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CONDUCTION THEORIES IN GASEOUS PLASMAS AND SOLIDS

FINAL REPORT

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I. SUMMARY OF RESEARCH EFFORT

The studies performed here were concerned with improving the correlation between theoretical prediction and experimental observation of the transport properties of ionized media. The microwave diagnostic techniques developed in a previous study (under AF49(638)687) were extended and brought to final completion. With this formulation it is possible to examine in some detail the properties of an anisotropic dielectric or plasma in a geometry which minimizes the effect of boundaries. In addition an attempt was made to ascertain the effect of a modified collision model on the electromagnetic properties of a partially ionized plasma in the presence of a dc magnetic field. The results, while of some theoretical interest, are of negligible practical importance - except possibly in the magnetohydrodynamic limit.

a) Interaction between electromagnetic waves and cold plasmas (L. Mower).

The following abstract of a recently published paper describes the content of the work completed on this project:

> "The interaction between a cylindrically symmetric anisotropic plasma column and bounded electromagnetic waves is analyzed theoretically. The properties of a cylindrical cavity coaxial with a cold plasma column and coaxial with a static magnetic field are determined. The shift in the resonant frequency of the cavity-plasma system is calculated in the high-electron density limit and compared with the numerical solution presented earlier."

b. Conductivity theory of partially ionized plasmas (L. Mower)

Most treatments of the conductivity of electron plasmas introduce a relaxation hypothesis to include the effect of electron-ion or electron-neutron particle short range collisions. Unless care is taken, this method violates conservation of particles and shows up as a basic inconsistency whenever polarization charge is present. To circumvent this, a number of different models have been introduced by Krook, Bhatnager, and Gross[1]. We have determined the conductivity for an electron gas in the presence of a dc magnetic field for a particular collision model which describes relaxation to a local equilibrium as distinct from relaxation to thermodynamic equilibrium (see Appendix 1). The resulting Fourier transform of the conductivity takes the following simple form

$$\sigma_T(\underset{\sim}{k},\omega) = \sigma(\underset{\sim}{k},\omega) - \frac{a(\sigma\cdot\underset{\sim}{k})(\underset{\sim}{k}\cdot\sigma)}{1 + a\,\underset{\sim}{k}\cdot\sigma\cdot\underset{\sim}{k}} ,$$

where σ is the conductivity calculated on the basis of relaxation to thermal equilibrium, $\underset{\sim}{k}$ is the propagation constant, and the constant a is given by

$$a = KT/nq^2\omega(\omega\tau+i) .$$

Here K is Boltzmann's constant, T is the absolute temperature of the unperturbed plasma (assumed to be Maxwellian), n is the plasma density, q is the charge of an electron,

ω is the radian frequency of the disturbance and τ is the effective relaxation time.

To begin with, we see that the conductivity tensor, σ_T, differs from σ only for $\underline{k} \neq 0$ and for $\underline{k} \cdot \sigma \neq 0$. That is, the effect of the modification in the collision model shows up only when polarization charge exists -the electron density within the plasma is both time and spatially dependent. Polarization charge exists or is generated within a plasma by electromagnetic waves whenever the direction of propagation is across a dc magnetic field, and the electromagnetic wave has a component of the $\underline{E}$ field along the direction of propagation - (extraordinary mode). For this mode the dependence of the propagation constant on temperature and the plasma properties has been calculated. The effect of the modification of the relaxation model on the properties of the propagation constant are slight at low temperatures and at high temperatures. At low temperatures the effect is most evident at the resonance frequency where it shows up as a slight broadening of the line of magnitude $1 + n^2 \kappa T/mc^2$ times the broadening which exists on the conventional relaxation model. (Here n^2 is the square of the index of refraction of the plasma at the half power points of the line, and is proportional to $\tau\omega_p^2/\omega$.) The concept of a "line" at resonance is only valid for low $\omega\tau$, a condition for which the modification in the relaxation model yields almost negligible effects.

At high temperature, the temperature dependence of σ , the conductivity of the usual relaxation model, is an order of magnitude more important than that arising from the correction to σ incorporated in σ_T . In addition, the concept of a hot partially ionized plasma itself is suspect.

The tentative conclusion drawn, then, is that the models proposed by Krook, Bhatnager, and Gross[1] to ensure microscopic conservation of charge in a partially ionized plasma are of interest from a standpoint of theory only and lead to negligible modification of the propagation properties of a plasma. It is conceivable that in the magnetohydrodynamics limit the modification will be significant.

Reference

1. N. Krook, P. C. Bhatnager, and E. Gross, Phys. Rev. 94, 511 (1954)

c) Magnetoacoustic effects in plasmas (L. Mower)

After the inception of this work, the following paper appeared on this subject, hence, this project has been suspended.

"Magnetoacoustic Effects in Nondegenerate Semiconductors," H. N. Spector, Phys. Rev. 125, 1880 (1962).

II. ADMINISTRATIVE NOTES

a) **Publications**

"Interaction between Cold Plasmas and Guided Electromagnetic Waves," Lyman Mower and S. J. Buchsbaum, Physics of Fluids 5, 1545 (1962).

b) **Public Lectures**

"Development of conduction theory in solids" - a lecture for Liberal Arts students - see Appendix 2.

c) **Personnel**

Professor L. Mower (principal investigator)

Mr. Richard Brooks (research assistant, summer 1962 only)

Mr. Brooks will continue in this field of physics as a candidate for a Ph. D. degree in Physics at the University of New Hampshire.

Appendix 1

CONDUCTIVITY THEORY IN A PARTIALLY IONIZED PLASMA

To study the dielectric properties of a plasma, or a metal, or a semi-metal, we represent the behavior of the large number of individual particles by that of a representative particle moving according to Newtonian mechanics. Thus if the material under study has a density of n particle per unit volume, then we approximate the actual motion of these particles by assuming an independent particle model in which the effects of the interparticle forces on any one particle are absorbed into an all pervading electromagnetic or acoustic field. We describe the number of particles that have velocity components between $\underset{\sim}{v}$ and $\underset{\sim}{v} + d\underset{\sim}{v}$ and position coordinates between $\underset{\sim}{r}$ and $\underset{\sim}{r} + d\underset{\sim}{r}$ by

$$f(\underset{\sim}{r}, \underset{\sim}{v}, t)\, d^3v\, d^3r,$$

where the distribution function $f(\underset{\sim}{r}, \underset{\sim}{v}, t)$ must satisfy the Boltzmann equation

$$\frac{\partial f}{\partial t} + \underset{\sim}{v} \cdot \nabla f + \frac{\underset{\sim}{F}}{m} \cdot \nabla_v f = \left(\frac{\partial f}{\partial t}\right)_c . \quad (1)$$

The particular form chosen for the expression representing the change in the number density per unit time arising from collisions, is motivated by the physical model describing the collisions and by mathematical tractability[1,2,3,4]. We introduce the following three assumptions:

1) the electron-neutral particle encounters are short range only and introduce no change in the number density.

2) the electrons scatter from neutral particles which are essentially at rest: hence the electrons scatter into an isotropic velocity space.

3) to include any possible polarization effects, the number density after scattering $n(r,t)$, may differ from that in the absence of any perturbing field.

Assumption (1) implies that he continuity equation for the electrons must be satisfied:

$$\frac{\partial n}{\partial t} + \nabla \langle n\underset{\sim}{v} \rangle = 0, \tag{2}$$

where

$$\langle nv \rangle \equiv \int f(\underset{\sim}{r},\underset{\sim}{v},t)\underset{\sim}{v}\, d\, v.$$

Hence, the integral of $(\partial f/\partial t)_c$ over all velocity space must vanish. The last two assumptions may be satisfied by the approximation[4]

$$(\partial f/\partial t)_c = -\left[f(\underset{\sim}{r},v,t) - f_s(\underset{\sim}{r},v,t) \right]/\tau, \tag{3}$$

where

$$f_s(\underset{\sim}{r},v\,t) = f_0\,(\underset{\sim}{r},v^2,t). \tag{4}$$

We assume that the plasma is close to thermal equilibrium and that the following linearization is valid:

$$
\begin{aligned}
f(\underset{\sim}{r}, \underset{\sim}{v}, t) &\sim f_0(v^2, 0) + f_1(\underset{\sim}{r}, \underset{\sim}{v}, t) \\
\underset{\sim}{E}(\underset{\sim}{r}, t) &\sim \qquad\qquad \underset{\sim}{E}_1(\underset{\sim}{r}, t) \\
\underset{\sim}{H}(\underset{\sim}{r}, t) &\sim \underset{\sim}{H}_0 + \underset{\sim}{H}_1(\underset{\sim}{r}, t) \\
n(\underset{\sim}{r}, t) &\sim n_0 + n_1(\underset{\sim}{r}, t),
\end{aligned}
\tag{5}
$$

where $f_0(v^2)$, the distribution function in the absence of any perturbing electric fields is isotropic. For simplicity it is taken to be the Maxwell-Boltzmann distribution function

$$
f_0(v^2) = n(m/2\pi kT)^{3/2} \exp(-mv^2/2kT), \tag{6}
$$

with this assumption, (4) is seen to be

$$
f_s(\underset{\sim}{r}, \underset{\sim}{v}, t) \sim f_0(v^2) + n_1(\underset{\sim}{r}, t)\, \partial f_0/\partial n, \tag{7}
$$

and (3) becomes

$$
(\partial f/\partial t)_c = -(f_1/\tau + n_1(\underset{\sim}{r}, t) f_0(v^2)/n_0\tau \tag{8}
$$

Hence, the Boltzmann equation simplifies to

$$
\frac{\partial f}{\partial t} + \underset{\sim}{v} \cdot \nabla f_1 - (q\mu_0/m)\, \underset{\sim}{v} \times \underset{\sim}{H}_0 \cdot \nabla_v f_1 + (f_1 - n_1 f_0/n_0)/\tau = (q/m)\, \underset{\sim}{E}_1 \cdot \nabla_v f_0(v^2) \tag{9}
$$

Upon multiplying equation (9) by $-q\, \underset{\sim}{v}^{(0,1)}$ and by $-q v^2$ and integrating over all velocity space, we obtain the continuity equation, the statement of conservation of momentum, and energy.

$$-q\,\partial n_1/\partial t + \nabla\cdot \underset{\sim}{j}_1 = 0 \tag{10a}$$

$$\frac{\partial \underset{\sim}{j}_1}{\partial t} - q\langle n\overrightarrow{\underset{\sim}{v}\underset{\sim}{v}}\rangle + \frac{\underset{\sim}{j}_1}{\tau} = \frac{q^2 n_0}{m}\underset{\sim}{E} + q\frac{\mu_0}{m}\underset{\sim}{j}_1\times\underset{\sim}{H}_0 \tag{10b}$$

$$\frac{\partial}{\partial t}\langle n\frac{mv^2}{2}\rangle + \nabla\cdot\langle n\frac{mv^2}{2}\underset{\sim}{v}\rangle = \left(\frac{3}{2}n_1KT - \langle n\frac{mv^2}{2}\rangle\right)/\tau . \tag{10c}$$

The modification of the collision term (3) ensures us of conservation of charge as well as energy. Its influence on the expression for the current is not apparent but arises in the relationship between the pressure tensor $\langle n\overrightarrow{\underset{\sim}{v}\underset{\sim}{v}}\rangle$, and the current.

The solution to (9) may be found by the method of integrating over a trajectory which is equivalent to the method of characteristics. It is found to be

$$f_1(\underset{\sim}{r},\underset{\sim}{v},t) = \int_0^\infty ds\, e^{-s/\tau}\left[\frac{2}{m}q\,\underset{\sim}{E}\cdot\underset{\sim}{v}\frac{df_0}{dv^2} + \frac{n_1}{n_1}f_0\right]' . \tag{11}$$

Here the prime denotes the fact that the variables $\underset{\sim}{r},\underset{\sim}{v},t$ in the integrand are to be replaced by $t' = t-s$, $\underset{\sim}{v}' = \mathbb{R}\cdot\underset{\sim}{v}$, and $\underset{\sim}{r}' = \underset{\sim}{r} - \int_0^s \mathbb{R}\,\underset{\sim}{v}\,dx$ Here $\mathbb{R}$ is a rotation operator, $\mathbb{R}(s) = \exp(\underset{\sim}{\omega}s X)$ with the matrix representation

$$\mathbb{R}(s) = \begin{bmatrix} \cos\omega_0 s & -\sin\omega_0 s & 0 \\ \sin\omega_0 s & \cos\omega_0 s & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

where

$$\underset{\sim}{\omega}_0 = -(q\mu_0/m)\underset{\sim}{H}_0 = \omega_0\,\underset{\sim}{z}$$

Before actually calculating the vector current density, we first obtain a pair of relationships which will allow us to eliminate n_1 . To do so, we introduce the Fourier transforms

$$\underset{\sim}{E}_1(\underset{\sim}{k},\omega) = (1/2\pi)^2 \iint \underset{\sim}{E}(\underset{\sim}{r},t)\, e^{-i(\underset{\sim}{k}\cdot\underset{\sim}{r}-\omega t)}\, d\underset{\sim}{r}\, dt$$

$$\underset{\sim}{E}(\underset{\sim}{r},t) = (1/2\pi)^2 \iint \underset{\sim}{E}_1(\underset{\sim}{k},\omega)\, e^{+i(\underset{\sim}{k}\cdot\underset{\sim}{r}-\omega t)}\, d\underset{\sim}{k}\, d\omega. \qquad (13)$$

Thus

$$f_1(\underset{\sim}{k},\underset{\sim}{v},\omega) = \int_0^\infty ds\, e^{-s/\tau} \iint \frac{d\underset{\sim}{r}\,dt}{(2\pi)^2}\left[\frac{2q}{m}\underset{\sim}{E}_1(\underset{\sim}{r}',t-s)\cdot\underset{\sim}{v}'\frac{df_0}{dv'^2} + \frac{n_1(\underset{\sim}{r}',t-s)}{\tau n_0} f_0\right] e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}+i\omega t} \qquad (14)$$

$$= \int_0^\infty ds\, e^{-s/\tau}\left[\frac{2q}{m}\underset{\sim}{E}_1(\underset{\sim}{k},\omega)\cdot\underset{\sim}{v}'\frac{df_0}{dv^2} + \frac{n_1(\underset{\sim}{k},\omega)}{\tau n_0}f_0\right] e^{-i\underset{\sim}{k}\cdot(\underset{\sim}{r}-\underset{\sim}{r}')+i\omega s},$$

where $\exp\, i\underset{\sim}{k}\cdot(\underset{\sim}{r}'-\underset{\sim}{r})$ is seen to be independent of $\underset{\sim}{r}$ when use is made of the definition of $\underset{\sim}{r}'$. We may rewrite (14) as

$$f_1(\underset{\sim}{k},\underset{\sim}{v},\omega) = \frac{2q}{m}\frac{df_0}{dv^2}\underset{\sim}{I}\cdot\underset{\sim}{E}_1(\underset{\sim}{k},\omega) + \frac{n_1(\underset{\sim}{k},\omega)}{\tau n_0}f_0 I, \qquad (15)$$

where

$$\left\{\begin{matrix}\underset{\sim}{I}\\ I\end{matrix}\right\} = \int_0^\infty e^{-s/\tau}ds \left\{\begin{matrix}\underset{\sim}{v}'\\ 1\end{matrix}\right\} e^{i\underset{\sim}{k}\cdot(\underset{\sim}{r}'-\underset{\sim}{r})+i\omega s}. \qquad (16)$$

An integration by parts of the expression for I yields the following relationship between I and $\underset{\sim}{I}$[4]:

$$i\,\underset{\sim}{I}\cdot\underset{\sim}{k} = \left[-1 + (1 - i\omega\tau) I\right]/\tau . \tag{17}$$

The Fourier transform of the vector current density

$$\underset{\sim}{j}_{1e}(\underset{\sim}{k},\omega) = -q \int \underset{\sim}{v}\, f_1(\underset{\sim}{k},\underset{\sim}{v},\omega)\, d\underset{\sim}{v} \tag{18}$$

is given by

$$\underset{\sim}{j}_{1e}(\underset{\sim}{k},\omega) = \sigma(\underset{\sim}{k},\omega)\cdot \underset{\sim}{E}_1(\underset{\sim}{k},\omega) - q\, n_1(\underset{\sim}{k},\omega)\underset{\sim}{R}(\underset{\sim}{k},\omega), \tag{19}$$

where

$$\sigma(\underset{\sim}{k},\omega) = -(2q^2/m)\int d\underset{\sim}{v}\,(df_0/dv^2)\,\overleftrightarrow{\underset{\sim}{v}\underset{\sim}{I}}(\underset{\sim}{k},\omega) \tag{20a}$$

and

$$\underset{\sim}{R}(\underset{\sim}{k},\omega) = \int d\underset{\sim}{v}\ \underset{\sim}{v}\, I(\underset{\sim}{k},\omega)\, f_0(v^2)/n_0\tau . \tag{20b}$$

The components of σ for $f_0(v^2)$ a Maxwellian distribution have been calculated and tabulated elsewhere.[5]

From (17) we find the following relation between $\underset{\sim}{R}$ and $\underset{\sim}{\sigma}\cdot\underset{\sim}{k}$

$$\begin{aligned} i\,\underset{\sim}{\sigma}\cdot\underset{\sim}{k} &= -\frac{2iq^2}{m}\int d\underset{\sim}{v}\ \frac{df_0}{dv^2}\left(\overleftrightarrow{\underset{\sim}{v}\underset{\sim}{I}}\right)\cdot\underset{\sim}{k} \\ &= -\frac{2q^2}{m}\int d\underset{\sim}{v}\ \frac{df_0}{dv^2}\ \underset{\sim}{v}\,(1-i\omega\tau)I/\tau \\ &= n_0 q^2 (1-i\omega\tau)\ \underset{\sim}{R}(\underset{\sim}{k},\omega)/KT, \end{aligned} \tag{21}$$

where we have made use of the Maxwellian form of $f_o(v^2)$.

Thus:

$$\underset{\sim}{R}(\underset{\sim}{k},\omega) = iKT\sigma\cdot\underset{\sim}{k}/n_o q^2(1-i\omega\tau), \tag{22}$$

and the current may be written as

$$\underset{\sim}{j}_1(\underset{\sim}{k},\omega) = \sigma(\underset{\sim}{k},\omega)\cdot\underset{\sim}{E}_1(\underset{\sim}{k},\omega) - \frac{iKTq\,n_1(\underset{\sim}{k},\omega)\,\sigma\cdot\underset{\sim}{k}}{n_o q^2(1-i\omega\tau)}. \tag{23}$$

We may introduce a further simplification by making use of the Fourier transform of the conservation relation:

$$-i\omega\left(-qn_1(\underset{\sim}{k},\omega)\right) + i\underset{\sim}{k}\cdot\underset{\sim}{j}_1(\underset{\sim}{k},\omega) = 0. \tag{24}$$

Thus, if we substitute this relation for n_1 into (23), we find

$$\underset{\sim}{j}_1 = \sigma\cdot\underset{\sim}{E}_1 + iKT(\underset{\sim}{k}\cdot\underset{\sim}{j}_1)(\sigma\cdot\underset{\sim}{k})/\omega n_o q^2(1-i\omega\tau), \tag{25}$$

which may be solved for $\underset{\sim}{k}\cdot\underset{\sim}{j}_1$ in terms of $\underset{\sim}{k}\cdot\sigma\cdot\underset{\sim}{k}$, i.e.

$$\underset{\sim}{k}\cdot\underset{\sim}{j}_1 = \underset{\sim}{k}\cdot\sigma\cdot\underset{\sim}{E}/\left[1-iKT(\underset{\sim}{k}\cdot\sigma\cdot\underset{\sim}{k})/n_o q^2\omega(1-i\omega\tau)\right] \tag{26}$$

Hence, the current density, (25) may be rewritten as

$$\underset{\sim}{j}_1(\underset{\sim}{k},\omega) = \sigma\cdot\underset{\sim}{E}_1 + \frac{iKT(\underset{\sim}{k}\cdot\sigma\cdot\underset{\sim}{E}_1)\sigma\cdot\underset{\sim}{k}}{n_o q^2\omega(1-i\omega\tau)-iKT\underset{\sim}{k}\cdot\sigma\cdot\underset{\sim}{k}}, \tag{27}$$

or as

$$\underset{\sim}{j}_1(\underset{\sim}{k},\omega) = \sigma_T(\underset{\sim}{k},\omega)\cdot\underset{\sim}{E}_1(\underset{\sim}{k},\omega), \tag{28}$$

where σ_T is defined by

$$\sigma_T(\mathbf{k},\omega) = \sigma(\mathbf{k},\omega) - \frac{[KT/n_0q^2\omega(\omega\tau+i)](\sigma\cdot\mathbf{k})(\mathbf{k}\cdot\sigma)}{1+[KT/n_0q^2\omega(\omega\tau+i)]\,\mathbf{k}\cdot\sigma\cdot\mathbf{k}}. \tag{29}$$

If, instead of the continuity equation for the electronic charge density, we had used Poisson's equation to eliminate n_1

$$i\mathbf{k}\cdot\mathbf{E}_1(\mathbf{k},\omega) = -q n_1(\mathbf{k},\omega)/\epsilon_0, \tag{30}$$

we would have found the following equation instead of (25)

$$\mathbf{j}_{1e}(\mathbf{k},\omega) = \sigma\cdot\mathbf{E}_1 - [KT\epsilon_0/n_0q^2(1-i\omega\tau)](\mathbf{k}\cdot\mathbf{E}_1)\,\sigma\cdot\mathbf{k} \tag{25}$$

This may be expressed as

$$\mathbf{j}_{1e}(\mathbf{k},\omega) = \sigma_{T'}\cdot\mathbf{E}_1(\mathbf{k},\omega), \tag{28'}$$

where $\sigma_{T'}$ is defined by

$$\sigma_{T'} \equiv \sigma - [iKT\epsilon_0/n_0q^2(\omega\tau+i)](\sigma\cdot\mathbf{k})\mathbf{k}. \tag{29'}$$

The equivalence of the two conductivities tensors σ_T and $\sigma_{T'}$ is realized by requiring the electric field and the current density to satisfy Maxwell's equations. The conductivity tensor σ satisfies Onsager's relations:

$$\sigma_{ij}(\mathbf{H}_0) = \sigma_{ji}(-\mathbf{H}_0); \tag{31}$$

hence, it is apparent from the symmetry of σ_T expressed in (29) that

$$\sigma_{T_{ij}}(\underset{\sim}{H_0}) = \sigma_{T_{ji}}(-\underset{\sim}{H_0}) \tag{32}$$

However, the fact that $\sigma_{T'}$ satisfies Onager's relations is not obvious from the form of (29'). They are only satisfied when the relationship between $\underset{\sim}{K}$ and $\sigma_{T'}$ found from Maxwell's equation is utilized. For this reason, when only an implicit knowledge of $\underset{\sim}{K}(\sigma_T)$ is to be used, σ_T is preferable to $\sigma_{T'}$.

In the consideration of the propagation of an electromagnetic wave in an infinite plasma we discuss separately the two cases of propagation parallel to the direction of the dc magnetic field, and propagation in a direction normal to the dc magnetic field.

Propagation Parallel to the External dc Magnetic Field

For this case there exist three independent modes: two transverse modes and one longitudinal mode. The two transverse modes describe circularly polarized waves with $\underset{\sim}{E}$ fields entirely in the plane transverse to the direction of propagation. Hence, $\sigma_T = \sigma$; i.e., no polarization charge exists and the electromagnetic properties of the plasma are unaltered by the modified collision term. The propagation properties of the longitudinal mode are altered by the modified collision term only the the extent that at high temperatures the index of refraction for $\omega_p^2/\omega^2 >> 1$ is entirely real: $n^2 = \omega_p^2 mc^2/\omega^2 KT$, $\omega_p^2 = n_0 q^2/m\epsilon_0$.

For low temperatures the index of refraction is

$$n^2 = (mc^2/\kappa T)(1+i/\omega\tau)^2(3-i/\omega\tau)^{-1}[\omega^2(1+i/\omega\tau)/\omega_p^2 - 1],$$

which differs little from the value found in the usual relaxation model.

Propagation Across the dc Magnetic Field

In examining the effect of the modified relaxation model for $\underset{\sim}{k} \perp \underset{\sim}{H}_0$, i.e., for propagation across the dc magnetic, we note first that the only mode of any interest is the extraordinary mode. For this mode the polarization charge is large near the shifted cyclotron frequency:

$$\omega^2 = \omega_b^2 + \omega_p^2 .$$

Near the zeros of $(\mathcal{E}_T)_{11} = 1 + i(\sigma_T)_{11}/\omega\epsilon_0$ for low temperature, $\kappa T/mc^2 \ll 1$ the width of the shifted cyclotron frequency line is broader over the usual collision broadening by a factor $1 + \kappa T\omega_p^2\tau/\omega mc^2$ which might be significant for dense plasmas. The difficulty with this approach is that the shape of the line is highly dependent on the electron density and collision frequency. Hence the broadening would be difficult to detect.

At high temperatures, the effects of the modified collision term on the electromagnetic properties of the plasma are of order $(k^2\kappa T/m\omega_b^2)^{1/2}$ smaller than that for the normal relaxation model. In addition, the concept of a partially ionized plasma at high temperatures is itself suspect.

For the dispersion relation for the propagation across the dc magnetic field $\underset{\sim}{k} \perp \underset{\sim}{H}_0$ and a purely longitudinal

electric field $\underset{\sim}{E} \parallel \underset{\sim}{k}$ reference is made to Lewis and Keller[3] in which corrections are given to the index of refraction stated above.

References

1) N. Krook, P. L. Bhatnagar, and E. Gross, Phys. Rev. 94, 511 (1954).

2) J. L. Warren and R. A. Ferrell, Phys. Rev. 117, 1252 (1960).

3) R. M. Lewis and J. B. Keller, Physics of Fluids 5, 1248 (1962).

4) M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. 117, 937 (1960).

5) L. Mower, Phys. Rev. 116, 16 (1959).

Appendix 2

DEVELOPMENT OF CONDUCTIVITY THEORIES IN METALS*

Introduction

In recent years considerable research has been devoted towards understanding the nature of conduction of electricity in metals. These investigations have yielded strong support for the independent particle model of normal metals. This model for the conduction of electricity in solids was suggested by Lorentz by comparing the conductivity process in metals with that in ionized gases. The model had suffered in the past from lack of a sound theoretical basis and yielded disturbing results when minor corrections were calculated to consider the effect of the interactions between the particles. Hence, when one realizes how simple the model is, it is surprising to note that within certain broad limits it is a valid model. The recent investigations have pointed out, in particular, in what way the model must be altered and in what way the particle contributing to the conductivity must be reinterpreted. To understand what

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*This lecture is largely based upon the following two articles and must not be interpreted as constituting original work.

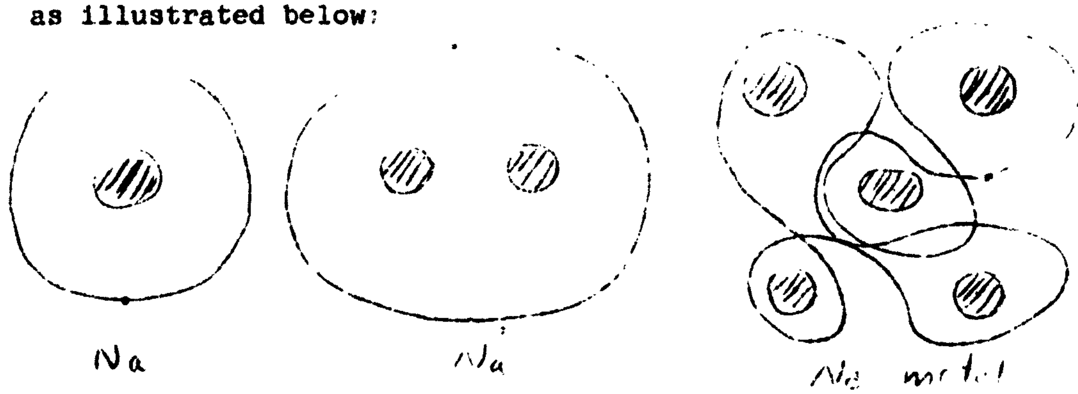
"Superconductivity and Production of High Magnetic Fields," E. Mendoza, The Times Science Review Winter 1962.

"Electrons in Metals: A Short Guide to the Fermi Surface," J. M. Ziman, Contemporary Physics 3, 241, 321, 401 (1962).

these changes mean for a normal metal, we present first a brief description of the classical Lorentz model, and its present reinterpretation. This is followed by a more detailed description of another phenomenon which also plays a role in the conduction of electrons in solids - superconductivity.

2. Lorentz model of a metal.

What characterizes a metal to most people is either the luster of a metal or its electrical properties, both of which are intimately connected. In order to appreciate the mathematical models used to describe the physical properties of metals it is useful to study the formation of a metal in terms of a collection of atoms. Thus, if the metal were sodium the formation of a metal from the atom might be visualized as proceeding through two stages as illustrated below:



(In addition to the closed shell of electrons surrounding the nucleus we have also depicted the single valence electron.) What is of interest to note is that in the formation of molecular sodium (a stable structure) each electron now spends about half its time about any given sodium nucleus. This is intensified in a solid where we see that each electron spends just as much of its time

around its "own" nucleus as around each of the neighboring nuclei. Inasmuch as there are eight nearest neighbors to any given sodium nucleus in metallic sodium we can no longer say that each electron is tied down to a given atom. Instead, we see that the electrons are free to wander throughout the metal and are not tied to a given nucleus.

On the basis of this picture we are led to postulate that a reasonable model of a metal might be that of a gas of free (non-interacting) electrons. In order to preserve macroscopic charge neutrality we introduce into the model an equal number of singly charged heavy ions which simply provide a background of positive charges. The electrical and optical properties of a metal then arise from the fact that on this model the electrons alone play a role -- positive ions serve simply to keep the entire gas from exploding. Such a collection is termed an electron plasma.

We should note that in this model of a metal we have ignored completely the influence of the heavy ions or the influence of the inner electrons other than that of charge neutralization. As a consequence, the above model is quite restricted and we should expect that an experimental test of it would show its limitations and its range of validity.

The model of a metal which we have just drawn is to be contrasted with a model for an insulator. A typical insulator is NaCl -- table salt. The atoms of sodium and chlorine differ in that the first has a single valence electron while the second has a vacancy in its outer shell and has a great affinity for electrons. In the process of formation of the molecule NaCl from

atomic sodium and chlorine the valence electron of sodium is lost almost completely to the chlorine; no sharing of the electron exists in this model - at least to the extent that it is possible in molecular sodium. When we pass to the comparison of metallic sodium with table salt the differences are more striking. The ability to share electrons which is characteristic of metals is almost completely lacking in table salt. Hence, we would not expect our model that we have just formulated, to apply to insulators.

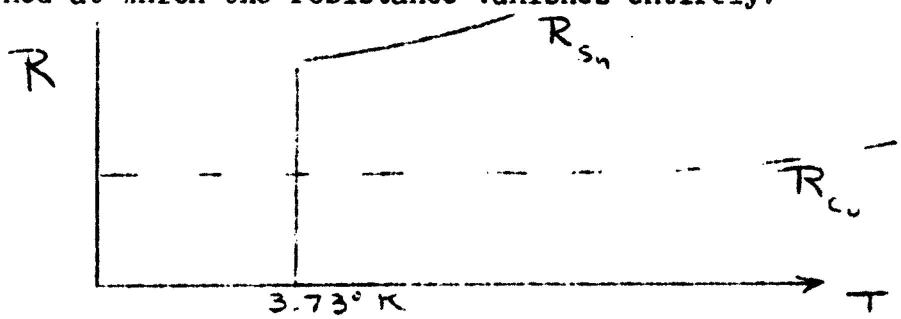
In comparing the predicted with the observed optical properties of metals the theoretical model is seen to be surprisingly good. One aspect of the model which is interesting is that if it is shock excited it oscillates just like jelly - this is also borne out in practice and is termed plasma oscillations. However, when we examine the observations in detail we notice discrepancies with the theoretical model. To understand them we examine again our theoretical model to see what has been left out. The picture we suggested as representing a metal was that of a free-electron gas - we neglected entirely the Coulomb interaction between the electrons and between the ions and electrons. While the interaction force between two isolated electrons is large, that in a plasma is quite weak. From the good correlation between theoretically and experimentally observed optical properties of metals we know that the plasma model has a certain range of validity. To improve upon it we must include the weak, short range (screened) electron-electron interaction in the model. To

do so, it is instructive to examine the motion of an electron as it moves through a plasma. If the plasma is dense enough we would "see" not a bare electron but an electron surrounded by a slight hole moving through the plasma. The electron pushes its neighboring electrons out of the way and attracts the positive ions to it. The combination of the "bare" electron plus its "clothing" of the absence of nearby neighboring electrons is termed a quasi-particle. If we rephrase the plasma model of a free-electron gas with a background of neutralizing positive charge to be a gas of charged quasi-particles instead, the agreement of theory with experiment is almost exact. This reinterpretation of the Lorentz or plasma model of a metal is due to Lev Landau and is one of the many theoretical predictions which have recently earned him the Nobel prize in physics.

The metals to which the modified Lorentz model - the Landau model - applies are in general very good conductors. The term normal metal is used to classify such conductors as Ag, Cu, etc. To show that the model has a limited range of validity and that there exist abnormal metals, one only has to examine the resistance of the metals as a function of temperature. Resistance is an electrical concept analogous to friction and describes the dissipative processes present in the material. In the absence of any resistance the response of a conductor to an impressed electric field would be infinite. Actually, in any real situation we know that the electrons on a metal encounter obstructions of one kind or another which impede their progress. These obstructions, impurities or lattice vibrations, absorb some of the

momentum, and at the same time some of the energy of the electrons which are responding to the impressed electric field. As the temperature is lowered we expect that the friction due to the lattice vibrations would decrease as the vibrations are temperature dependent. However, we do not expect the friction to vanish altogether as there are always some impurity atoms present.

A comparison of our expectations with experiment yields some surprising results. Copper, an excellent conductor at room temperature, becomes a somewhat better conductor at low temperatures. The resistance of copper decreases by about a hundred-fold or a thousand-fold from its value at room temperature. By low temperatures we mean a few degrees above absolute zero. If we examine the resistance of tin as a function of temperature, we see that it also decreases by a similar factor until 3.73° K is reached at which the resistance vanishes entirely.



Thus, tin, which is a poor conductor at ordinary temperatures, becomes a superconductor at or below 3.73° K. To give an idea of the numbers involved, copper, which is one of our best conductors near room temperature, is a perfect insulator relative to tin at or below the transition temperature, 3.73° K of tin.

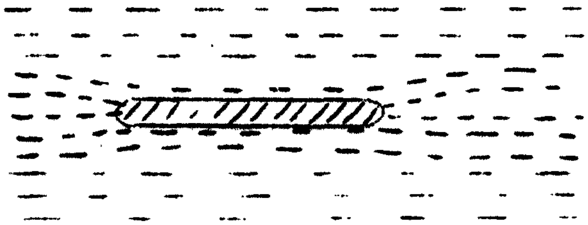
The theoretical description we've given above for the resistance of a metal obviously fails when applied to a superconductor.

while the resistance due to lattice vibrations is expected to decrease with temperature, a finite residual resistance due to impurities is expected in all physical samples. We find, however, that even "dirty" tin is a superconductor. From this fact, we anticipate that an explanation of superconductivity in terms of the electrical properties alone would be impossible.

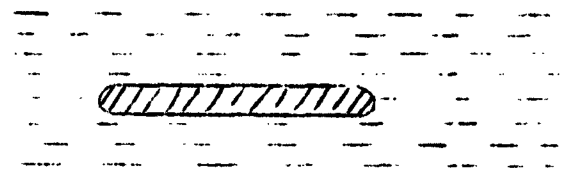
3. Magnetic Phenomena of Superconductors

Kamerlingh Onnes, the first to liquefy helium (1908) used the technique he had developed to obtain such low temperatures to study the properties of metals. In 1911 he discovered the startling fact that at 4.2° K the resistance of mercury decreased abruptly to zero. He further discovered that in the presence of a sufficiently strong magnetic field the superconductivity property of mercury was destroyed.

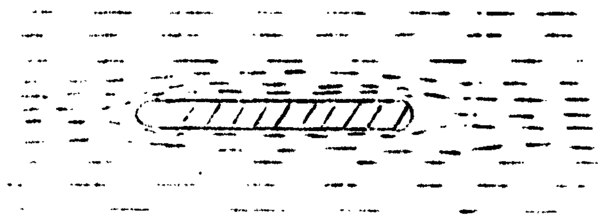
It took some twenty-two years of sorting conflicting data and improving experimental technique before the next significant experiment on superconductors was made. In 1933 Meissner and Ochsenfeld showed that a superconductor showed perfect diamagnetism and that hence the superconducting properties of a metal were intimately connected to their magnetic properties. To illustrate the behavior of a perfect diamagnet we compare a bar magnet (ferromagnet), a non-ferromagnet like copper, and a perfect diamagnet when each is placed in a magnetic field. To visualize the field lines we might make use of iron filings to trace out the magnetic field lines.



The pattern of lines near a bar magnet is such that they tend to be pulled in or concentrate on the poles of the magnet.



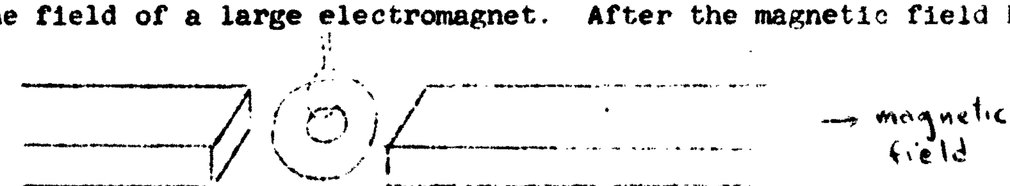
On the other hand, a non-ferromagnetic metal is unaffected by and in turn does not affect an external magnetic field.



A superconductor, however, exhibits perfect diamagnetism, that is, it excludes the magnetic field lines.

The application of a sufficiently strong magnetic field destroys the superconducting properties and hence the Meissner effect. In such a high field the material which behaved like a perfect diamagnet behaves like a non-ferromagnet metal. The removal of the strong magnetic field restores the superconducting properties and the metal again expels the magnetic field lines.

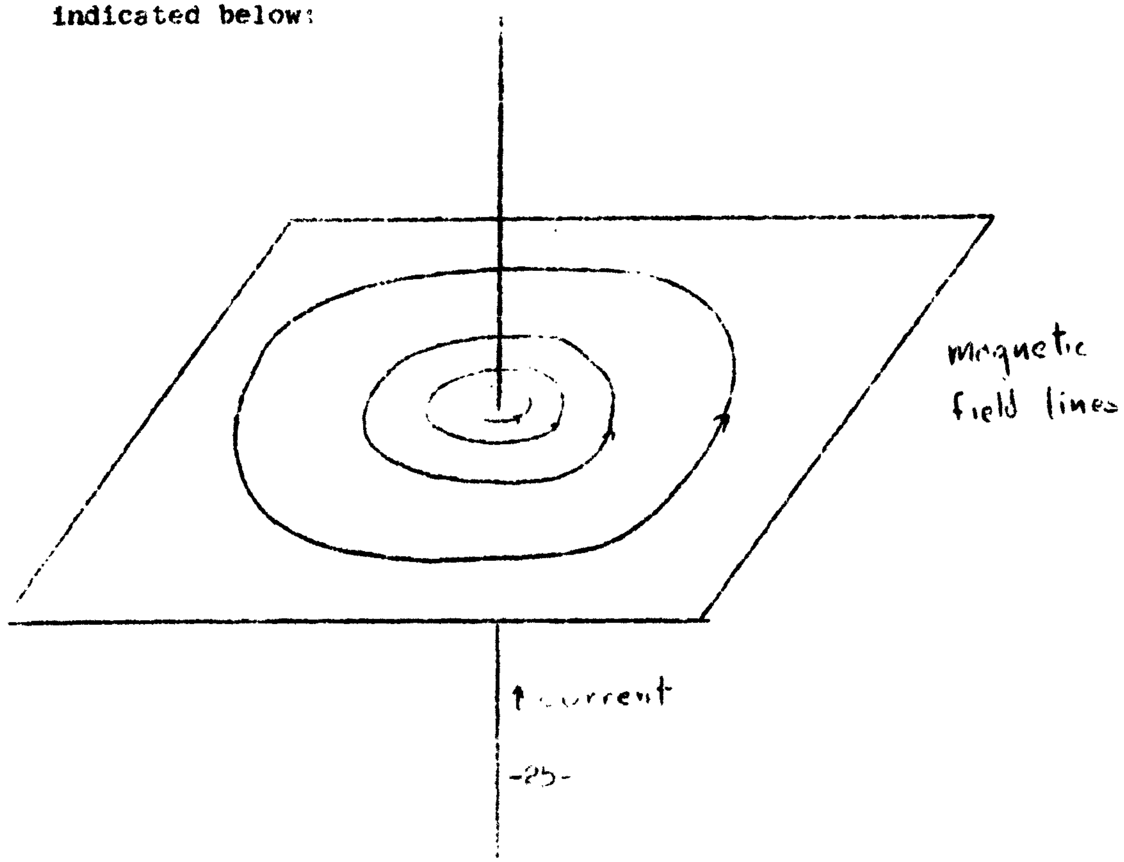
An interesting experimental phenomenon which was hitherto inexplicable is that of persistent currents - an experiment first performed in 1923 by Kamerlingh Onnes and W. Tuyn. The experiment may be schematized as follows. A ring of metal is suspended in the field of a large electromagnet. After the magnetic field has



been applied, the ring is removed. In the removal process eddy currents are set up in the ring which provides a closed path for the current to flow. The resultant current loop may be detected

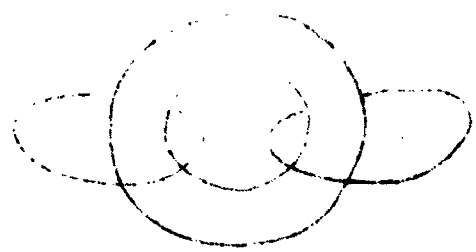
by its associated magnetic field. For normal metals the lifetime of such circulating currents before they decay to nothing is of the order of a second. For superconductors, the lifetime of such currents is infinite. In fact, superconducting rings have been maintained for a period of several years without any measureable attenuation of the current.

To indicate how this penemenon is understandable in terms of the Meissner effect it is worth while to consider Oersted's measurements in 1820 on the influence of electricity on small magnetized needles - compasses. We may interpret this experiment most simply by again having recourse to the iron filings. If the iron filings are distributed evenly over a sheet of paper through which a wire is passed, then when current is allowed to flow the iron filings trace out the magnetic field lines as indicated below:



The interpretation given to this experiment becomes more interesting if performed for different values oft he current passing through the wire. We infer that we may imagine the field lines as completely elastic - yet never breaking. They expand or contract to zero as the current - and hence the field - is increased or decreased to zero.

These statements are true for normal metals and enable us to interpret the phenomenon of persistent ring currents. Thus, when the ring is removed from the field of the electromagnet eddy currents arise which give rise to circulating currents. Tied to these currents is of course a magnetic field (which is depicted only in one plane in the accompanying figure). In



normal metals dissipation processes are present which tend to decrease the current with a resultant collapse of the magnetic field. The magnetic field in the case of normal metals collapses down through the metal to nothing.

Not so in the case of superconductors. When a superconductor is withdrawn from an electromagnet circulating currents again are formed with their associated magnetic fields. In this case, however, because of the Meissner effect the magnetic field is excluded from the interior of the wire and is confined to the exterior space. Even though dissipative forces might exist, the

magnetic field lines cannot collapse to zero. Hence, the lifetime of a circulating current in an ideal superconductor is infinite.

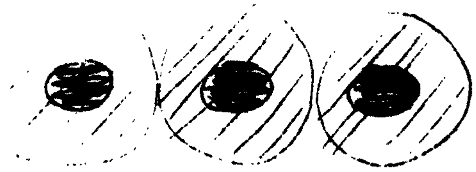
4. Theoretical Treatment

In searching for a theoretical explanation for superconductivity we again look to the simple plasma model with the hope that by including the interelectron force some insight towards understanding the penomenon may be found. To begin with we expect that the forces involved will be weak - much weaker than a coulomb force. Otherwise, the thermal vibrations of the lattice would not destroy superconductivity at such low temperatures and we would observe the phenomenon at more reasonable temperatures. Secondly, it cannot be an electrical phenomenon but must be largely a magnetic phenomenon. That is, the explanation of the Meissner effect should explain to a large degree the properties of superconductors.

To describe the results of the microscopic theory it is of considerable value to introduce a macroscopic analogy. Consider the motion of a boat on a lake. As it moves it develops a wake which affects the motion of any other boat on the lake. In a similar fashion we may consider the motion of an airplane which also develops a wake which affects the motion of any nearby airplanes. This time, however, we cannot see the wake but must trust to other senses.

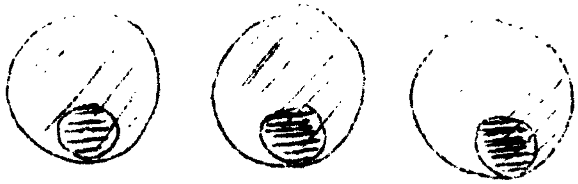
In direct analogy to this consider the passage of an electron through a solid. As it passes a heavy ion it pushes

the outer electron shell away and tends to draw the positive



unpolarized ions
with circumambulating
electrons

core towards it. In other words, it polarizes the medium as indicated in the figure below:



polarized ions
after the passage
of an electron

Another electron moving in the opposite direction finds that the wake of the first electron - the polarization wake - is an easier road to travel than over the plasma. Hence, there is a net correlation between electrons with equal but opposite moments and spin. As a consequence we also see that this is a source for a possible attraction between two electrons.

Of course, if the ion clouds are tightly held, the effect is much smaller than the screened coulomb force and is negligible. This is the case with normal metals. For metals with a large number of electrons and which polarize easily the effect can become dominant. This then is the explanation of superconductivity. We describe its properties not in terms of a collection of quasi-particles but in terms of a collection of pairs of particles each pair having a net zero momentum.

5. Flux Quantization

So far we have made no reference to the wave aspect of electrons. Indeed, it is from this point of view that one of the most fruitful -to theoretical physics - predictions of the theory of superconductivity arises. To begin with from the experiments by Davisson and Germer we are led to attribute a wave-like nature to electrons: if the "electron" has a given velocity then we infer that its wavelength varies inversely to this velocity (de Broglie relationship). In a normal metal in the absence of an impressed electric field the velocities of the electrons are distributed at random in direction but each has a magnitude close to the Fermi velocity. In the presence of an impressed electric field, the directions of the velocities of the electrons are distributed not quite at random so that a net current results. In both cases associated with the spectrum of velocities is a spectrum of wavelengths.

In the case of a superconductor, however, in the absence of any net current flow the particles condense into pairs each having a net zero momentum. Thus the wavelength spectrum of the particles may be reinterpreted in terms of the pairs where we see that only one wavelength is present - one of infinite length. In the presence of a finite current the particles again pair off, now however each pair has a net momentum - the cancellation is not perfect. In fact, all the pairs have the same net momentum. Hence, the wavelength spectrum as opposed to a normal metal instead of having a distribution of possible wavelengths has only one - that associated with the momentum common to all the condensed pairs.

The rather unusual wavelength dependence of the condensed pairs of a superconductor show up when we consider the phenomenon called flux quantization. Because of the topological difference between a simply connected object like a solid cylinder and a multiply connected object like an anular ring the flux threading the ring is spatially quantized. This can actually be observed by performing careful measurements on very small superconducting cylindrical rings, (Doll and Nabauer in Munich, 1961; Deaver and Fairbanks at Stanford, 1961).

This fact, the spatial quantization of the magnetic flux on a macroscopic scale, provides the physicist an example of a quantum phenomenon which had hitherto only been observed at a microscopic level.

Resumé

In a somewhat cursory manner we have seen the development of the Lorentz or plasma model of a metal. This mathematical model has been used to predict various electrical properties of metals; these predictions in turn were compared with observations in order to obtain a measure of the range of validity of the model. For a class of metals - good conductors - the normal metals, the correlation between theory and experiment was greatly improved by including the interactions of the electrons with each other and with the positive charges. This was achieved by reinterpreting the model as that of a collection of quasi-particles - each quasi-particle consisting of a "bare" electron with the "clothing" of its interactions around it.

For another class of metals - the superconductors - the correlation between theory and experiment was improved by making a much more drastic change. The model was replaced not by a collection of quasi-particles but by a collection of pairs. The resulting theory while not yielding a precise confirmation of experimental results yet served as a very good step towards a more complete theory.